**Topic**: Infinite/essential discontinuities

**Question**: Choose the correct statement.

**Answer choices**:

A An infinite discontinuity exists at a point when the right-hand and left-hand limits both approach $+\infty$, or both approach $-\infty$.

B An infinite discontinuity exists at a point when the right-hand limit approaches $-\infty$ while the left-hand limit approaches $+\infty$, or vice versa.

C An infinite discontinuity exists at a point when the right-hand limit is $0$ and the left-hand limit approaches $+\infty$ or $-\infty$, or vice versa.

D All of the above are true.

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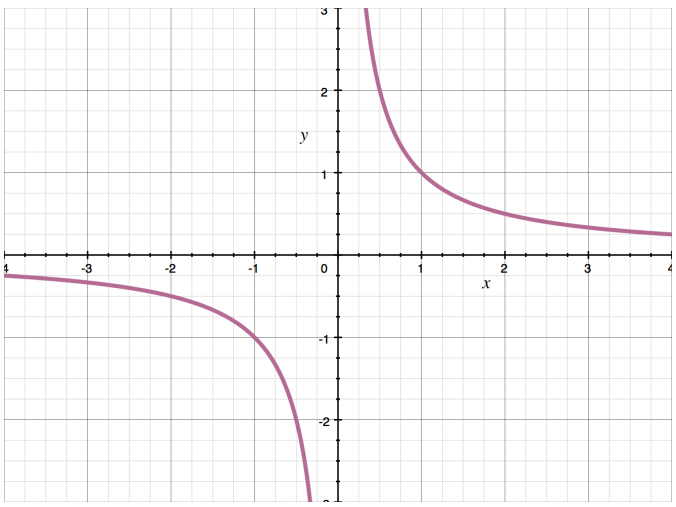
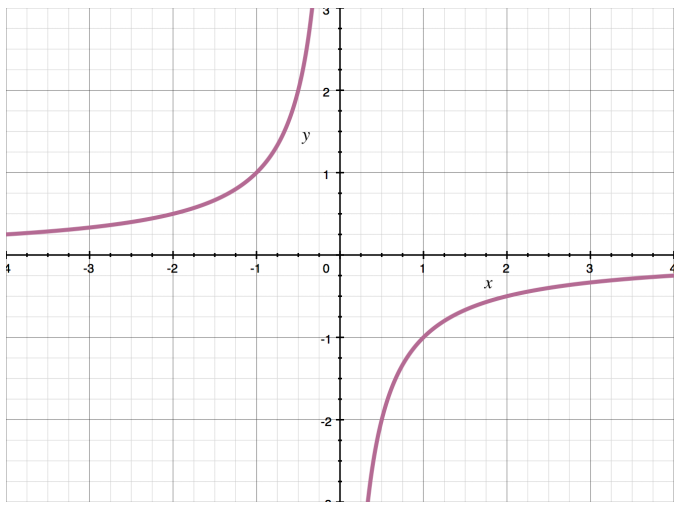
**Solution**: D

An infinite discontinuity exists at a point when the right-hand and left-hand limits both approach $-\infty$, or both approach $+\infty$.

Depending on who you ask, answer choice A may or may not be considered correct. Some mathematicians consider $\pm\infty$ to be a limit, such that, if both the left- and right-hand limits are $+\infty$, then the general limit is also $+\infty$, and there is no discontinuity in the function at all. Or if both the left- and right-hand limits are $-\infty$, then the general limit is also $-\infty$, and there is no discontinuity in the function at all. Others believe that a limit can only be a finite value. In that case, even if both the left- and right-hand limits are equal to $+\infty$, or both are equal to $-\infty$, the general limit still does not exist.

This kind of discontinuity also exists at any point where the right-hand limit approaches $-\infty$ while the left-hand limit approaches $+\infty$, or vice versa. In a case like that, the graph might look like either of these:




Infinite discontinuities even exist where one of the one-sided limits approaches a finite value while the other one-sided limit approaches an infinite value, so answer choice C is also true.

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**Topic**: Infinite/essential discontinuities

**Question**: Choose the correct description of the infinite discontinuity.

$$f(x) = \frac{1}{x}$$

**Answer choices**:

A The function has an infinite discontinuity at $x = 1$.

B The function has an infinite discontinuity at $x = -\infty$.

C The function has an infinite discontinuity at $x = 0$.

D The function has an infinite discontinuity at $x = +\infty$.

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**Solution**: C

To solve for the discontinuities of the function, start by evaluating the right-hand and left-hand limits as $x$ approaches $0$. We need to use $0$ for the limit because the function does not exist when $x = 0$.

For the right-hand limit, we'll use $3$, $2$ and $1$.

$$\lim_{x \to 0^+} \frac{1}{x} = \frac{1}{3} = 0.3$$

$$\lim_{x \to 0^+} \frac{1}{x} = \frac{1}{2} = 0.5$$

$$\lim_{x \to 0^+} \frac{1}{x} = \frac{1}{1} = 1$$

For the left-hand limit, we'll use $-3$, $-2$ and $-1$.

$$\lim_{x \to 0^-} \frac{1}{x} = \frac{1}{-3} = -0.3$$

$$\lim_{x \to 0^-} \frac{1}{x} = \frac{1}{-2} = -0.5$$

$$\lim_{x \to 0^-} \frac{1}{x} = \frac{1}{-1} = -1$$

We can see that the right-hand limit is $+\infty$ and the left-hand limit is $-\infty$. This means that there's an infinite discontinuity at $x = 0$.

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**Topic**: Infinite/essential discontinuities

**Question**: Choose the correct description of the infinite discontinuity.

$$f(x) = \frac{x}{x-4}$$

**Answer choices**:

A The function has an infinite discontinuity at $x = 4$.

B The function has an infinite discontinuity at $x = -\infty$.

C The function has an infinite discontinuity at $x = -4$.

D The function has an infinite discontinuity at $x = +\infty$.

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**Solution**: A

To solve for the discontinuities of the function, start by evaluating the right-hand and left-hand limits as $x$ approaches $4$. We need to use $4$ for the limit because the function does not exist when $x = 4$.

For the right-hand limit, we'll use $7$, $6$ and $5$.

$$\lim_{x \to 4^+} \frac{x}{x-4} = \frac{7}{7-4} = \frac{7}{3} = 2.3$$

$$\lim_{x \to 4^+} \frac{x}{x-4} = \frac{6}{6-4} = \frac{6}{2} = 3$$

$$\lim_{x \to 4^+} \frac{x}{x-4} = \frac{5}{5-4} = \frac{5}{1} = 5$$

For the left-hand limit, we'll use $1$, $2$ and $3$.

$$\lim_{x \to 4^-} \frac{x}{x-4} = \frac{1}{1-4} = \frac{1}{-3} = -0.3$$

$$\lim_{x \to 4^-} \frac{x}{x-4} = \frac{2}{2-4} = \frac{2}{-2} = -1$$

$$\lim_{x \to 4^-} \frac{x}{x-4} = \frac{3}{3-4} = \frac{3}{-1} = -3$$

We can see that the right-hand limit is $+\infty$ and the left-hand limit is $-\infty$. This means that there's an infinite discontinuity at $x = 4$.

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